THE USE OF INNOVATIVE TEACHING METHODS FOR 'MAXIMISING' THE ENJOYMENT FROM LEARNING MATHEMATICAL CONCEPTS

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Abstract

This paper explores methods of bridging the gap between a basic mathematical background and the ability to learn and use more advanced techniques. It is demonstrated how students with an average (say, High School) level of mathematics skills can learn Dynamic Programming in an enjoyable manner. The approach presented has proven to be an easy to learn and easy to use practical method.

The main purpose of this paper is to find out if the use of innovative teaching and learning approaches (including multimedia and web-enhanced systems) contribute to students' learning and success in the assessments.

The presented method of teaching and learning has resulted in allowing an opportunity for the average student to learn a relatively advanced topic successfully. It has also made learning a mathematical topic an enjoyable experience for students.

Keywords: Multimedia, Web-Enhanced, Interactive, Dynamic Programming, Analogy, Recursive

Introduction

Teaching mathematical concepts to students who do not have a very strong background in mathematics is always challenging. This experience is also extremely rewarding when these students begin enjoying the journey, which leads to mastering the concepts. This paper will demonstrate how we can make it possible for learners with an average secondary education level of mathematical skills to make the transition to a more advanced level more successfully. The main purpose is to determine if the use of innovative methods of teaching (including the technologies associated with modern computing) enhances learning.

The crossing from a basic mathematical background to a more advanced level may be described as achieving the goal state from an initial state by reducing the distance between them.

Therefore, we may regard the entire problem as a goal programming exercise in which a number of objectives have to be attained simultaneously. It should be noted that we are not dealing with a rigorous goal programming exercise. The author is simply using the general ideas of goal programming as an analogy.

The goals of this goal programming exercise can be, in some cases, incompatible or in conflict with each other. Some of these goals include: minimising the time taken to learn and, hopefully, master the topic, maximising the enjoyment from learning, minimising the general fear associated with a mathematical topic, maximising students' perception of the practical uses of the topic, and minimising the distance between theory and practice.

Obviously, some of these objectives are more important than the others. Therefore, they may be ranked according to their priorities. For instance, it would be quite logical to place 'minimising the time taken to learn and hopefully master the topic' and 'maximising enjoyment' at the top of the list. These rankings can be incorporated into the problem as in a pre-emptive goal programming exercise. Of course, some subjectivity is bound to enter into our formulation.

In order to achieve our goals, we need to establish certain facts and figures. Based on this information, we may prepare our plan and strategy.

Foundation of the investigation

It was observed by the author (1996 – 2000) that the first half of a lecture on a mathematical subject is the most crucial part in terms of students' learning, and feelings about the topic. This study was carried out by recording the number of questions (by different 1st year tertiary students) relating to difficulties in grasping the concepts.

It was concluded that if well over 50% of the students do not grasp the basics during the first half of the lecture, a general lack of interest in the subject might be created. This lack of interest in the topic may even last for the entire semester.

A survey of the Toowoomba High School students' learning preferences, has re-confirmed that a large proportion (about 40 %) of students do not regard mathematics to be an enjoyable subject. See Figure 1.

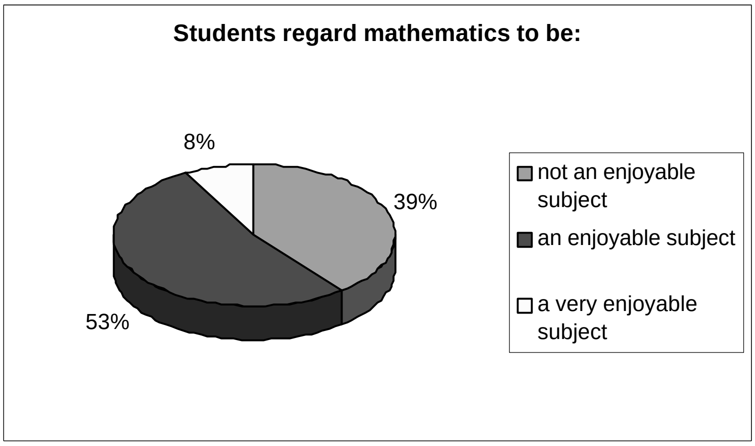


Figure 1 – Students' perception of mathematics

This survey, which will be referred to as THSS (Toowoomba High Schools Survey), was part of a research project funded by the Faculty of Business, USQ, 2002. Its main purpose was to investigate year 12 students' learning style preferences in statistics and related topics.

According to the THSS, 67% of the students who do not enjoy mathematics are those who do not have anyone in their immediate family with a strong mathematics background.

Given the high percentage that does not enjoy maths, it would be a reasonable strategy to focus on them when designing tertiary teaching materials. For instance, identifying these students' learning style preferences and considering them in the design of learning and teaching packages would be a move in the right direction. An investigation of the THSS students who do not enjoy mathematics has shown that:

- 51% prefer seeing many graphs, images and relevant pictures during the lessons;

- 44% prefer that the teacher provides a great deal of verbal explanations during the lesson/s; and

- only 5% prefer having the opportunity to read the topic from a book or handouts and then ask the teacher any questions they may have.

Further analysis has revealed that 73% of the THSS 'non-enjoyers' prefer to be told exactly what to do. In other words, a vast majority of students in this category may not have a preference for the constructivist methods of learning! So, how do they like to learn? The THSS has demonstrated that the use of other media in addition to text and providing clear direction would certainly benefit those students who do seem to enjoy mathematics.

Therefore, minimisation of the time taken in conveying the main message of the topic and maximising enjoyment from learning would be reasonable high priority objectives. Methods of dealing with this issue, to a certain extent, depend on the teacher and teaching materials available.

Let us select the topic of Dynamic Programming (DP) for the purpose of testing our stated hypothesis.

Based on teaching Dynamic Programming to students from a non-mathematical background, the author has developed a practical and general-purpose approach to formulating and solving DP problems. This approach uses a minimal amount of mathematical symbols, notations, equations and expressions in formulation and solution. The general-purpose approach was implemented as an interactive multimedia system in the late 90s.

Prior to the introduction of this general-purpose approach, only about 65% of the students attempted the Dynamic Programming questions in the examination. This figure did not seem to be a satisfactory level. It should be noted that this figure increased to well over 95% in the late 90s and continues to remain at that level.

The next section provides a foundation to the dynamic programming concepts adopted in the general-purpose approach.

Dynamic Programming

Dynamic programming (DP), closely associated with Bellmann (1957), is a sequential or multistage approach to formulating and solving mathematical programming problems.

The following references include some of the classic introductory material to Dynamic Programming, its concept and applications: Bellmann and Dreyfus (1962); Kaufmann and Cruon (1967) and Norman (1972). For a more modern but still classic problem (finding the ideal partner) see Smith (1997).

Dynamic programming can sometimes be the only feasible optimisation technique applicable to management and decision making problems. The fact, however, that no general formulation of dynamic programming has been available and each problem must be solved uniquely, has made this technique rather less attractive for practitioners to use. As suggested by Moores (1986, pp 967-969), dynamic programming is considered an intellectually appealing way of formulating a problem, but not a very useful way for solving it.

It is contended that, a general-purpose method of formulating and solving dynamic programming problems will help practically oriented students to learn, adopt and apply the technique to appropriate problems. The next section introduces an easy to learn and general-purpose method of formulating and solving Dynamic Programming problems. The main purpose is to demonstrate how our learning goals (mentioned earlier) can be achieved through diverse methods of teaching.

The General-Purpose Method and how it works

The general-purpose method relies on a tabular approach by adopting a General Purpose Table (GPT) and a Generalised Recursive Formula (GRF). The table and the formula are used as a template for different problems. (Nooriafshar 1992).

GPT contains information for every **stage** (a point or period for making decisions) of the following variables. This information is presented in a tabular format:

- the **input states** (conditions within the stages)
- the **possible decisions** for the stage and their values
- the resulting **output states** associated with each possible decision
- the **optimal decision** for the stage and its value

The definition of GRF is generalised and expanded as follows:

The value of the optimal decision for state i at stage n is the optimum (minimum or maximum) value of the decision resulting from leaving state i and entering state j separated by an appropriate operation (ie. +, x, -, @, etc, let us denote it by an asterisk), depending on the problem, from the value of the optimal decision for state j at stage n+1. It should be noted that state i is at stage n and state j is at stage n + 1. It is necessary to mention that the above statement forms the basis of a general purpose Recursive Formula which may be shown as:

Value of Optimal Decision$_n$ (i) = Optimum {Value of optimal decision$_n$ (ij) * value of optimal decision$_{n+1}$ (j)}

After presenting the above formula, it is pointed out to students that the second part of the right-Hand-Side of the equation is the recurrence of the Left-Hand-Side at an incremented stage. To reinforce the meaning of recursion, the author makes reference to his ‘Between-the-Two-Mirrors’ experiment as an analogy. Students are asked to visualise themselves standing between two facing mirrors and looking at their reflection reflected several times through the mirrors.

Finally, students, learn that they may, very easily, customise and use the same generic expression for different problems.

Conveying the message about the concept and meaning of the terms **stages** and **states** to the students is also an important step. The following analogy by the author has proven to be an effective way of getting the message across:

Imagine that you have $50 in your pocket/purse. You like buying the latest CD albums of your favourite artists. Assuming that a CD costs $25, you enter the Big W department Store.

At this stage the students are asked to comment on their financial status (**state**). The answer should be: at **stage** Big W the ‘entering-the-shop’ or the **input state** (financial status upon entering the shop) is $50. ‘What are the possible **decision/s** one may make’ is the next question. Most students should now be able to find out the right answer of:

One may purchase 0,1 or 2 CDs.

There is a very important message here with regard to the **decision**. The author deliberately guides the students towards finding out that the decision really depends on the ‘entering-the-shop’ (input) state of $50. If for instance, it is decided to buy 1 CD, then the ‘leaving-the-shop (output) state will be $25 (50-25). Similarly, other output states of $50 and $0 are other possibilities. The students are now told that they decide to enter another shop (for instance K Mart). So, the output state of $25 at previous stage (Big W) now becomes the input state into the stage n+1 (K Mart in this case). Similarly, the decision on what one may do (how many CDs to purchase) is again dependent on this input state.

Analogies such as the ones above contribute to the minimisation of learning time and learning process enjoyment objectives. They may also be implemented as interactive multimedia animations, which will provide the same opportunities to the distance mode students.

The next section demonstrates how a problem is approached using the general-purpose method. It should be noted that it is not intended to provide a tutorial on how to formulate and solve the problem. For these details, refer to Nooriafshar (1992). The main purpose is to demonstrate the fact that by adopting the general-purpose approach, one is able to produce the optimum solution in a single compact table.

An Example of How the General-purpose Method Works

Let us look at a classic problem:

X Y Z Ltd. produces a piece of electronic equipment comprising 3 modules: **alpha (A), beta (B) and gamma (C)**. All three modules must function for the system to operate successfully. The design team has pointed out that by connecting more (up to 3) modules in parallel, the reliability of the system is improved. There would be extra costs however, associated with this. The management would like to determine how many units of each module should be connected in order to achieve the maximum reliability, subject to not exceeding the system construction cost of $50.

Before delving into various concepts and mathematical equations, students are encouraged to form small groups, discuss the problem and propose practical solutions. This phase would form the basis of a constructivist approach in which the learner is guided towards finding out the solution. (Nooriafshar 2001).

It is interesting to note that according to the Toowoomba High Schools Survey (THSS), 52% of students prefer having the opportunity of being guided to find the solution.

As one can see, the best results for our problem would be achieved by connecting as many modules as possible (3 units of each) in parallel.

Since the cost of 3 units of each module exceeds the allocated cost of $50, then the most reliable system cannot be the answer. "Now, what about having a system with one unit of each module?" Students are asked. "Well, although this combination is financially allowed, it is the lowest reliable system." Students are informed. The final question in the process of guiding the students to seek the right answer is: "So, what do you think the answer should be?" At this stage, students are encouraged to think about what was presented earlier and then make a suggestion. Often, they conclude that the right answer must be somewhere between the two extremes.

The reliability and costs of connecting 1,2 or 3 modules in parallel are as shown in Figure 2:

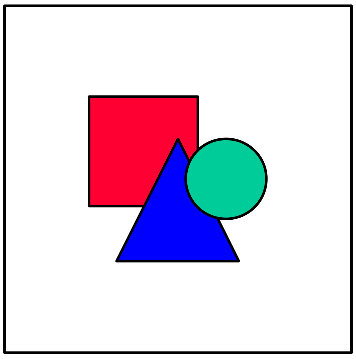
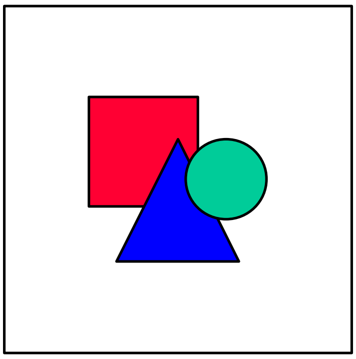
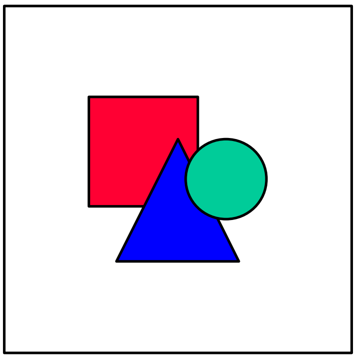
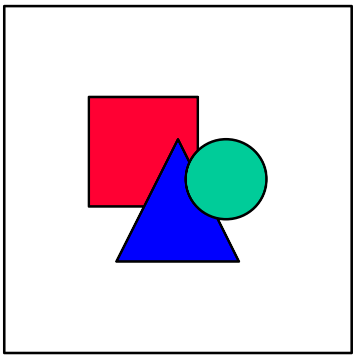
No Of Modules In Parallel	Reliability Of The Module (%) A B C	Cost Of Installing The Module ($) A B C
1	80  8 2  85	10  16  17
2	88 90 95	12 18 23
3	92 94 97	15 20 25

Figure 2 – Reliability and cost information

Therefore, we have to search the problem space (between the two extremes) and find the best alternative. Complete enumeration is a possibility, but is definitely not recommended as listing all the possibilities and calculating the costs would be an extremely tedious and time-consuming task. Hence, we would need to apply an appropriate partial enumeration technique such as dynamic programming.

This is a very important conclusion as it is the basis of finding an optimum solution within a set of possible solutions in many similar problems. Therefore, it is emphasized to students that understanding this general concept will help with most optimisation techniques.

Now, students are shown how to apply the general-purpose approach and formulate the Stages, States, Decision and the Recursive Relationship as follows:

Stages: The Type of Module (A, B and C)

States: The Amount Of Money Available (Left) To Allocate

Decision: How Many of Each Module to Connect (based on how much is available)

Using GRF, the **Recursive Relationship** is:

Value of Optimal Decision$_n$ (i) = Max {Value of optimal decision$_n$ (ij) x value of optimal decision$_{n+1}$ (j)}

Using GPT, the solution for the above problem is shown in Figure 3. As the figure illustrates, maximum probability of 0.7038 is achieved when 3, 2 and 1 units of A, B and C respectively are connected in parallel. See the solutions under the "Optimal Decision" column in Figure 2. Students are shown how to start with a blank table and complete it in step-by-step and systematic manner by linking output from a current stage to the next stage.

Stage	Input State	Possible Decisions	Optimal Decision	Output State	Possible Decision Value	Optimal Decision Value
C	24	1		7	.85	
		2	2	1	.95	.95
	22	1	1	5	.85	.85
	20	1	1	3	.85	.85
	19	1	1	2	.85	.85
	18	1	1	1	.85	.85
	17	1	1	0	.85	.85
	15	INFEAS	ABLE			
B	40	1		24	.82X.95=.779	
		2		22	.90X.85=.765	
		3	3	20	.94X.85=.799	.799
	38	1		22	.82X.85=.697	
		2		20	.90X.85=.765	
		3	3	18	.94X.85=.799	.799
	35	1		19	.82X.85=.697	
		2	2	17	.90X.85=.765	.765
		3		15	INFEASIBLE AT PREVIOUS STAGE	
A	50	1		40	.80X.799=.6392	
		2		38	.88X.799=.7031	
		3	3	35	.92X.765=.7038	.7038

Figure 3 – The solution table

It should be noted that students usually master using the table after performing 2 to 3 similar examples. This is confirmed by a very high (around 95%) rate of success in the examination. The next section describes how this method of teaching was implemented on computer as a web-enhanced interactive multimedia system.

Extension to a Web-Enhanced Multimedia System

The extension of the general-purpose method to a web-enhanced multimedia system was the outcome of a research project funded by the Faculty of Business, USQ in the late 90s.

Development of the web-enhanced multimedia system was based on the following beliefs:

Firstly, a multimedia system should not be regarded as a substitute for the traditional teaching/learning methods.

Secondly, a multimedia system should be able to enhance the existing teaching/learning materials. In other words, it ought to offer students different and additional features such as the opportunity of interacting with animations of the concepts as and when they wish. However, it must be remembered that the content and teaching/learning methods should still be the basis of any multimedia system. A large number of video and audio clips, and clickable icons and menu items do not necessarily add value.

The author also believes that the most effective multimedia system is still the actual teacher. That is the teacher who uses his/her hands and facial expressions, changes the tone of the voice and establishes eye contact with almost everyone during a lecture. Therefore, such a teacher utilizes and relies on the medium of body language to convey subtleties in a non-verbal manner too.

One of the main features of the web-enhanced multimedia system is its ability to facilitate the teaching of complex concepts of dynamic programming via specially designed animations and simulations.

Various illustrations, quizzes and reminders are used throughout this interactive learning tool. Illustrative examples use sound and animations to allow the learner to interact with the system.

Students are able to interact with the animations and investigate different situations. Hypertext links to explanations and links between various sections of the material are also amongst the features of the system. For details of the web-enhanced system, see:

http://www.usq.edu.au/users/mehryar/51349/dp/show001.htm

This multimedia system has made it possible to incorporate different learning style preferences such as Visual, Aural, Read/Write and Kinesthetic (VARK) noted by Fleming (1995) into the distance mode students learning packages. Therefore a simulation of face-to-face multimodal presentation, to a certain extent, is now available for all students. It should be mentioned that the use of graphics, animations and visual features in general, is a preferred option by 55% of the THSS students.

Therefore, all students, regardless of their geographical location and means of interaction with the University, may enjoy that extra level of explanation, which is usually conveyed during a traditional face-to-face lecture or tutorial situation.

Conclusions

The most interesting and, certainly satisfying, outcome was the performance of the students in the assessments. In the late 90s when the multimedia system was used for the first time, well over 95% achieved satisfactory results. The Cut-offs for grades were about 10 to 15 percent higher than the previous years too. These achievements and performances support the hypothesis that by adopting innovative ideas more students will be able to cross the bridge and learn advanced topics in mathematics in an enjoyable manner. A higher participation rate in the assessments indicates that the problem of the fear of mathematics has also been addressed.

The positive responses of the students also demonstrate that the multimedia extension is an effective means of reinforcing the learning process, particularly for those students who are not able to take advantage of the traditional (face-to-face) mode of delivery.

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